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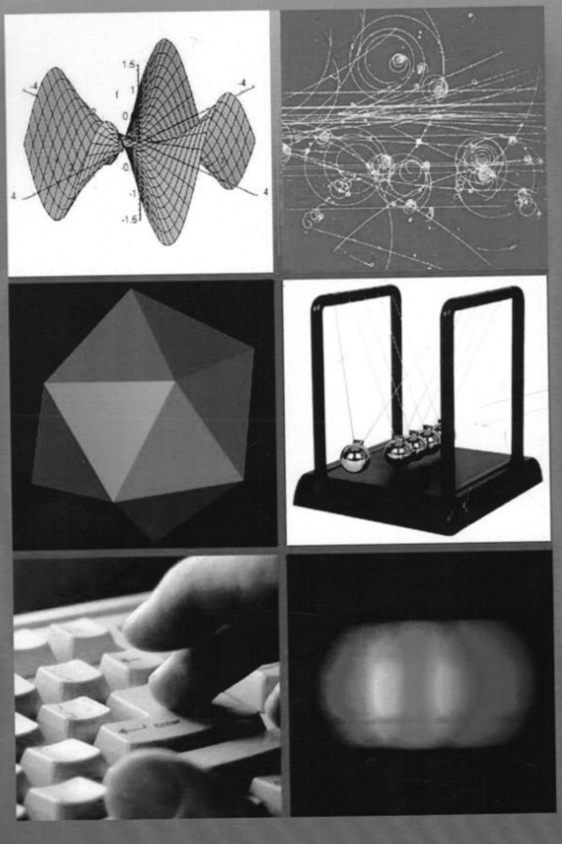

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Return problem of acoustics and its conditional correctness

Abstract. In this article the dynamic return task for acoustics equation is considered. For research of property of the operator derivative Frechet and the operator interfaced to it we will reduce differential statement of the return problem of acoustics to an operator look. At the beginning we consider the return problem of acoustics in case of the concentrated source, then we reduce an initial task to system of the nonlinear integrated equations of Voltaire of the second sort. In the return a task we will enter a new variable and we receive the return task in which according to additional information it is necessary to find the solution and acoustic rigidity of the environment. The received return task will be more preferable than initial statement for several reasons. And still, the return task can be reduced to system of the nonlinear integrated equations of Voltairian type for which it will be possible to receive a series of results, including theorems of a correctness and of convergence of a method of iterations of Landveber. Further the equation forms system of the nonlinear integrated equations of Voltaire of the second sort. We will reduce the return task for acoustics equation to an operator to a look further.
Key words: Return problem, acoustics equation, Voltaire nonlinear integrated equations, Landveber gradient method.

Introduction

The dynamic return task for acoustics equation is considered. For application of a gradient method of Landveber, it is developed computing methods of the solution of a nonlinear return problem of acoustics. We prove conditional stability of the decision of system of the nonlinear equations of Voltaire [1] and we define stability constants (Lipschitz's constants).

Definition 1 (Class of solutions of the return task).

We will say that $\sigma(x) \in \Sigma(l, M_\sigma, c_0, \rho_0, \sigma_*)$, if $\sigma(x)$ meets the following conditions:

$$\sigma(x) \in H^1(0,l), \ \|\sigma\|_{H^1(0,l)} \le M_\sigma,$$
$$0 < \sigma_* \le \sigma(x), x \in (0,l), \ \sigma_0 = c_0 \rho_0.$$

Definition 1 (Class of basic data). We will say that $g \in G(l, \beta, \gamma)$, if g meets the following conditions:

$$g \in H^1(0,2l), \ \|g'\|^2_{L_2(0,2l)} \le \beta, \ g(+0) = -\gamma,$$

Objects and methods of researches: We will assume that for $g^{(1)}, g^{(2)}$ from a class $G(l, \beta, \gamma)$ exist $\sigma^{(1)}, \sigma^{(2)} \in \Sigma(l, M_\sigma, c_0, \rho_0, \sigma_*)$, satisfying to the return task

$$u_{tt}^{(j)}(x,t) = u_{xx}^{(j)}(x,t) - \frac{(\sigma^{(j)})'(x)}{\sigma^{(j)}(x)} u_x^{(j)}(x,t), \quad x > 0, t > 0, \tag{1}$$

$$u^{(j)}(x,t)|_{t<0} \equiv 0, \tag{2}$$

$$u_x^{(j)}(+0,t) = \gamma \delta(t), \quad t > 0, \tag{3}$$

$$u^{(j)}(+0,t) = g^{(j)}(t), \quad t > 0. \tag{4}$$

for j = 1 and j = 2.

Considering that $s^{(j)}(x) = -\gamma\sqrt{\sigma^{(j)}(x)/\sigma^{(j)}(+0)}$, we will designate

$$q_1^{(j)}(x,t) = u_x^{(j)}(x,t),\ q_2^{(j)}(x) = \frac{1}{s^{(j)}(x)},\ q^{(j)}{}_3(x) = 2\frac{\left(s^{(j)}\right)'(x)}{s^{(j)}(x)}.$$

$$f_1^{(j)}(x,t) = \left[\left(g^{(j)}\right)'(t+x) - \left(g^{(j)}\right)'(t-x)\right],\ f_2^{(j)} = -\frac{1}{\gamma},\ f_3^{(j)}(x) = -\frac{2\left(g^{(j)}\right)'(2x)}{\gamma},\quad j = 1, 2. \tag{5}$$

As $\sigma^{(j)} \in \Sigma(l, M_\sigma, c_0, \rho_0, \sigma_*)$, $j = 1, 2$, that owing to designations can be estimated

$$\left\|q^{(j)}\right\|_{\bar{L}2(l)} \le M_{q},\ j = 1, 2,$$

where $M_q = M_q(l, M_\sigma, \gamma, \beta, \sigma_*)$. Let vector – function $q = (q_1, q_2, q_3)^T$, satisfies to system (6)

$$q(x,t) + Bq(x,t) = f(x,t), (x,t) \in \Delta(l). \tag{7}$$

Solution of a task Aq = f is supposed, but is claimed that there is the only steady decision for the data from the vicinity which are precisely set that is restriction on noise in entrance data is imposed.

For a conditional correctness of the considered task, unlike the similar theorem in [2] in the theorem below at a conclusion of the demanded constant in the main inequality not the assessment of a vector of q, but estimates was used

$$\|q_1\|^2_{L_2(\Delta(l))} \le 2l\exp\left\{\frac{2lM_\sigma^2}{\sigma_*^2}\right\}\beta, \quad \|q_2\|^2_{L_2(0,l)} \le \frac{2l}{\gamma^2}\exp\left\{\frac{lM_\sigma^2}{2\sigma_*^2}\right\}, \quad \|q_3\|^2_{L_2(0,l)} \le \frac{1}{\sigma_*^2}\|\sigma\|^2_{H_1(0,l)} = \frac{1}{\sigma_*^2}M_\sigma^2.$$

each of its component q_1, q_2, q_3. In work [2] in calculations the norm of everyone components was estimated through norm of a vector of q as the assessment of norm of a vector of q is the sum of estimates its component. And in this article as it is told, above, each estimates of its component were already used directly q_1, q_2, q_3.

The theorem. We will assume that for $f^{(i)} \in \bar{L}_2(l)$, $j = 1,2$, there are solutions of the return task $q^{(j)} \in \bar{L}_2(l)$, $j = 1,2$

$$q^{(j)}(x,t) + Bq^{(j)} = f^{(j)}(x,t),\ j = 1,2,\ (x,t) \in \Delta(l).$$

Then

$$\left\|q^{(1)} - q^{(2)}\right\|^2_{L_{2(l)}} \le C_1\left\|f^{(1)} - f^{(2)}\right\|^2_{\bar{L}_{2(l)}} \tag{8}$$

Here (9)

$$C_1 = \left(15 + \frac{50M_\sigma^2}{\gamma^2\sigma_*^2} + \frac{5lM_\sigma^2}{\sigma_*^2}\left(1 + \frac{5M_\sigma^2}{\gamma^2\sigma_*^2}\right)\exp\left\{\frac{lM_\sigma^2}{2\sigma_*^2}\right\}\right)\times$$

$$\times \exp\left\{\left(3l + 5\beta + \frac{25l}{\gamma^2}\right)\frac{M_\sigma^2}{\sigma_*^2} + \left(\frac{l^2}{\gamma^2} + \frac{25l^2 M_\sigma^2}{2\gamma^2\sigma_*^4} + \frac{10l\beta}{\gamma^2}\right)\exp\left\{\frac{lM_\sigma^2}{2\sigma_*^2}\right\} + \right.$$

$$\left. + \left(10\beta l \frac{M_\sigma^4}{\sigma_*^4} + 5l\beta + \frac{70l\beta}{\gamma^2}\right)\exp\left\{\frac{2lM_\sigma^2}{\sigma_*^2}\right\} + \frac{55l^2\beta M_\sigma^2}{\gamma^2\sigma_*^2}\exp\left\{\frac{5lM_\sigma^2}{2\sigma_*^2}\right\}\right\} \quad (10)$$

Proof. We will enter

$$\widetilde{q}(x,t) = (\widetilde{q}_1(x,t)\widetilde{q}_2(x)\widetilde{q}_3(x)) = q^{(1)}(x,t) - q^{(2)}(x,t),$$

$$\widetilde{f}(x,t) = (\widetilde{f}_1(x,t)\widetilde{f}_2(x)\widetilde{f}_3(x)) = f^{(1)}(x,t) - f^{(2)}(x,t),$$

$$(\widetilde{f})'(x) = (f^{(1)})'(x) - (f^{(2)})'(x).$$

Then from (8) follows

$$\widetilde{q}(x,t) = Bq^{(1)}(x,t) - Bq^{(2)}(x,t) = \widetilde{f}(x,t), (x,t) \in \Delta(l).$$

We will estimate the first to a component $\widetilde{q}_1$:

$$|\widetilde{q}_1(x,t)| = |\widetilde{f}_1(x,t)| + \frac{1}{2}\|\widetilde{q}_3\|(x) \times \left[\sqrt{\int_0^x |q_1^{(1)}(\xi, t+x-\xi)|^2 d\xi} + \sqrt{\int_0^x |q_1^{(1)}(\xi, t-x+\xi)|^2 d\xi}\right] +$$

$$+ \frac{1}{2}\|q_3^{(2)}\|(x)\left[\sqrt{\int_0^x \widetilde{q}_1^{(1)}(\xi, t+x-\xi)d\xi} + \sqrt{\int_0^x \widetilde{q}_1^{(2)}(\xi, t-x+\xi)d\xi}\right].$$

Considering that $\left(\sum_{i=1}^n a_i\right)^2 \le n\left(\sum_{i=1}^n a_i^2\right)$ for $a_i \ge 0$, receive

$$|\widetilde{q}_1(x,t)|^2 \le 5|\widetilde{f}_1(x,t)|^2 + \frac{5}{4}\|\widetilde{q}_3\|^2(x) \times \int_0^x \left[|q_1^{(1)}(\xi, t+x-\xi)|^2 + |q_1^{(1)}(\xi, t-x+\xi)|^2\right]d\xi +$$

$$+ \frac{4}{4}\|q_3^{(2)}\|(x)\int_0^x \left[\widetilde{q}_1^2(\xi, t+x-\xi) + \widetilde{q}_1^2(\xi, t-x+\xi)\right]d\xi. \quad (11)$$

We have

$$\|\widetilde{q}_1\|^2(l,x) \le 5\|\widetilde{f}_1\|^2(l,x) + \frac{5}{4}\int_0^x \int_\xi^{2l-\xi} \left\{\|\widetilde{q}_3\|^2(\xi) \times \int_0^\xi \left[|q_1^{(1)}(\xi', \tau+\xi-\xi')|^2 + |q_1^{(1)}(\xi', \tau-\xi+\xi')|^2\right]d\xi' \times\right.$$

$$\left.\times \|q_3^{(2)}\|^2(\xi)\int_0^\xi \left[\widetilde{q}_1^2(\xi', \tau+\xi-\xi') + \widetilde{q}_1^2(\xi', \tau-\xi+\xi')\right]d\xi'\right\} d\tau d\xi$$

$$\le 5\|\widetilde{f}_1\|^2(l,x) + \frac{5}{2}\int_0^x \|\widetilde{q}_3\|^2(\xi)\|q_1^{(1)}\|^2(l,\xi)d\xi + + \frac{5}{2}\int_0^x \|q_3^{(2)}\|^2(\xi)\|\widetilde{q}_1\|^2(l,\xi)d\xi. \quad (12)$$

We will estimate the second to a component:

$$|\widetilde{q}_2(x)| \le \frac{1}{2}\int_0^x \left|q_3^{(1)}(\xi)\widetilde{q}_2(\xi)\right| d\xi + \frac{1}{2}\int_0^x \left|q_2^{(2)}(\xi)\widetilde{q}_3(\xi)\right| d\xi \le \frac{1}{2}\left\|q_3^{(1)}\right\|(x)\left\|\widetilde{q}_2\right\|(x) + \frac{1}{2}\left\|\widetilde{q}_3\right\|(x)\left\|q_2^{(2)}\right\|(x).$$

Therefore,

$$\left\|\widetilde{q}_2\right\|^2(x) \le \frac{1}{2}\int_0^x \left(\left\|q_3^{(1)}\right\|^2(\xi)\left\|\widetilde{q}_2\right\|^2(\xi) + \left\|\widetilde{q}_3\right\|^2(\xi)\left\|q_2^{(2)}\right\|^2(\xi)\right) d\xi.$$

Estimate $\widetilde{q}_3$:

$$|\widetilde{q}_3(x)| \le \left|\widetilde{f}_3(x)\right|\left(1 + \gamma\left|B_2 q^{(2)}\right|\right) + \sum_{j=1}^{4}\omega_j(x),$$

where

$$\omega_1(x) = \gamma\left|(f_3^{(1)})(x)\right|\left|B_2 q^{(1)} - B_2 q^{(2)}\right|,\ \omega_2(x) = \frac{2}{\gamma}\left|B_4 q^{(1)} - B_4 q^{(2)}\right|,$$

$$\omega_3(x) = \left|B_2 q^{(1)}\right|\gamma\omega_2(x),\ \omega_4(x) = 2\left|B_4 q^{(2)}\right|\left|B_2 q^{(1)} - B_2 q^{(2)}\right|.$$

First, we have

$$\omega_1(x) \le \frac{\gamma}{2}\left|(f_3^{(1)})(x)\right|\left(\left\|q_3^{(1)}\right\|(x)\left\|\widetilde{q}_2\right\|(x) + \left\|\widetilde{q}_3\right\|(x)\left\|q_2^{(2)}\right\|(x)\right).$$

Secondly,

$$\omega_2(x) \le \frac{2}{\gamma}\int_0^x \left|\widetilde{q}_3(\xi)q_1^{(1)}(\xi,2x-\xi) + q_3^{(2)}(\xi)\widetilde{q}_1(\xi,2x-\xi)\right| d\xi$$

$$\le \frac{2}{\gamma}\left[\left\|\widetilde{q}_3\right\|(x)\sqrt{\int_0^x \left|q_1^{(1)}(\xi,2x-\xi)\right|^2 d\xi} + \left\|q_3^{(2)}\right\|(x)\sqrt{\int_0^x \widetilde{q}_1^{\,2}(\xi,2x-\xi) d\xi}\right].$$

Thirdly,

$$\omega_3(x) \le \left|B_2 q^{(1)}\right|\gamma\omega_2(x) \le \frac{\gamma}{2}\omega_2(x)\int_0^x \left|q_3^{(1)}(\xi)q_2^{(1)}(\xi)\right| d\xi \le \frac{\gamma}{2}\omega_2(x)\left\|q_3^{(1)}\right\|(x)\left\|q_2^{(1)}\right\|(x).$$

Fourthly,

$$\omega_4(x) \le \left\|q_3^{(2)}\right\|(x)\sqrt{\int_0^x \left|q_1^{(2)}(\xi,2x-\xi)\right|^2 d\xi}\ + \left(\left\|q_3^{(1)}\right\|(x)\left\|\widetilde{q}_2\right\|(x) + \left\|\widetilde{q}_3\right\|(x)\left\|q_2^{(2)}\right\|(x)\right).$$

In view of $\left(\sum_{i=1}^{n}|a_i|\right)^2 \le n\sum_{i=1}^{n}|a_i|^2$, receive

$$\widetilde{q}_3^{\,2}(x) \le 10\widetilde{f}_3^{\,2}(x)(1 + \gamma^2\left|B_2 q^{(2)}\right|^2) + \frac{5\gamma^2}{2}\left|f_3^{(1)}(x)\right|^2\left(\left\|q_3^{(1)}\right\|^2\left\|\widetilde{q}_2\right\|^2(x) + \left\|q_2^{(2)}\right\|^2\left\|\widetilde{q}_3\right\|^2(x)\right) +$$

$$+ \frac{20}{\gamma^2}\left(\left\|\widetilde{q}_3\right\|^2(x)\int_0^x \left|q_1^{(1)}(\xi,2x-\xi)\right|^2 d\xi + \left\|q_3^{(2)}\right\|^2\int_0^x \left|q_1^{(2)}(\xi,2x-\xi)\right|^2 d\xi\right) \times \left(1 + \frac{\gamma^2}{4}\left\|q_3^{(1)}\right\|^2\left\|q_2^{(1)}\right\|\right) +$$

$$+10\left\|q_3^{(2)}\right\|^2\int_0^x \left|q_1^{(2)}(\xi,2x-\xi)\right|^2 d\xi\left(\left\|q_3^{(1)}\right\|^2\left\|\widetilde{q}_2\right\|^2(x) + \left\|q_2^{(2)}\right\|^2\left\|\widetilde{q}_3\right\|^2(x)\right).$$

Thus, we have

$$\left\|\widetilde{q}_3\right\|^2(x) \le 10\left(1 + \frac{\gamma^2}{4}\left\|q_3^{(2)}\right\|^2\left\|q_2^{(2)}\right\|^2\right)\left\|\widetilde{f}_3\right\|^2(x) + + \frac{5\gamma^2}{2}\int_0^x \left|f_3^{(1)}(\xi)\right|^2\left(\left\|q_3^{(1)}\right\|^2\left\|\widetilde{q}_2\right\|^2(\xi) + \left\|q_2^{(2)}\right\|^2\left\|\widetilde{q}_3\right\|^2(\xi$$

$$+\frac{20}{\gamma^2}\left(\int_0^x \|\widetilde{q}_3\|^2(\xi)\int_0^\xi \left|q_1^{(1)}(\zeta,2\xi-\zeta)\right|^2 d\zeta d\xi+\frac{1}{2}\left\|q_3^{(2)}\right\|^2\|\widetilde{q}_1\|^2(l,x)\right)\times\left(1+\frac{\gamma^2}{4}\left\|q_3^{(1)}\right\|^2\left\|q_2^{(1)}\right\|^2\right)+$$

$$+10\left\|q_3^{(2)}\right\|^2\int_0^x\left(\left\|q_3^{(1)}\right\|^2\|\widetilde{q}_2\|^2(\xi)+\left\|q_2^{(2)}\right\|^2\|\widetilde{q}_3\|^2(\xi)\right)+\int_0^\xi \left|q_1^{(2)}(\zeta,2\xi-\zeta)\right|^2 d\zeta d\xi.$$

For convenience we will designate

$$p_1(x)=\|\widetilde{q}_1\|^2(l,x),\quad p_j(x)=\|\widetilde{q}_j\|^2(x),\quad j=2,3,\quad x\in(0,l).$$

We will enter designations

$$\mu_1=\left(10+\frac{5\gamma^2}{2}\left\|q_3^{(2)}\right\|^2\left\|q_2^{(2)}\right\|^2\right),\ \ \mu_2=\left(\frac{10}{\gamma^2}+\frac{5}{2}\left\|q_3^{(1)}\right\|^2\left\|q_2^{(1)}\right\|^2\right)\left\|q_3^{(2)}\right\|^2,$$

$$\mu_3(\xi)=\frac{5\gamma^2}{2}\left|f_3^{(1)}(\xi)\right|^2\left\|q_3^{(1)}\right\|^2+10\left\|q_3^{(2)}\right\|^2\left\|q_3^{(1)}\right\|^2\int_0^\xi\left|q_1^{(2)}(\zeta,2\xi-\zeta)\right|^2 d\zeta,$$

$$\mu_4(\xi)=\frac{5\gamma^2}{2}\left|f_3^{(1)}(\xi)\right|^2\left\|q_2^{(2)}\right\|^2+\left(\frac{20}{\gamma^2}+5\left\|q_3^{(1)}\right\|^2\left\|q_2^{(1)}\right\|^2\right)\int_0^\xi\left|q_1^{(1)}(\zeta,2\xi-\zeta)\right|^2 d\zeta+$$

$$+10\left\|q_3^{(2)}\right\|^2\left\|q_2^{(2)}\right\|^2\int_0^\xi\left|q_1^{(2)}(\zeta,2\xi-\zeta)\right|^2 d\zeta, \tag{18}$$

We will enter function

$$P(x)=(5+\mu_1+5\mu_2)\left\|\widetilde{f}\right\|^2+\int_0^x\sum_{j=1}^3 k_j(\xi)p_j(\xi)d\xi, \tag{19}$$

Therefor [3]

$$\frac{P'(x)}{P(x)}\le\sum_{j=1}^3 k_j(x),$$

and, applying Gronuoll's inequality, we will receive

$$p(x)\le P(x)\le P(0)\exp\left\{\int_0^x\sum_{j=1}^3 k_j(\xi)d\xi\right\}.$$

On the other hand, considering equality

$$\int_0^x\int_0^\xi \widetilde{q}_1^{\,2}(\xi',2\xi-\xi')d\xi'd\xi=\frac{1}{2}\int_0^x\left(\int_{\xi'}^{2x-\xi'}\widetilde{q}_1^{\,2}(\xi',\zeta)d\zeta\right)d\xi'=\frac{1}{2}\|\widetilde{q}_1\|^2(l,x)$$

have

$$k_1\le\frac{5M_\sigma^2}{2\sigma_*^2}\left(1+\frac{10}{\gamma^2}+\frac{5lM_\sigma^2}{\gamma^2\sigma_*^2}\exp\left\{\frac{lM_\sigma^2}{2\sigma_*^2}\right\}\right),$$

$$\int_0^x k_2(\xi)d\xi\le\left(\frac{l}{2}+5\beta\right)\frac{M_\sigma^2}{\sigma_*^2}+10\beta l\frac{M_\sigma^4}{\sigma_*^4}\exp\left\{\frac{2lM_\sigma^2}{\sigma_*^2}\right\}$$

Putting estimates we will receive

$$\|\tilde{\sigma}\|^2_{H^1(0,l)} \le C\|g^{(1)} - g^{(2)}\|^2_{H^1(0,2l)},$$

where $C = C_2(l+1)$.

Results and their discussion: Theorem 1.5. Let for $g^{(1)}, g^{(2)}$ from a class $G(l,\beta,\gamma)$ exist $\sigma^{(1)}, \sigma^{(2)} \in \Sigma(l, M_\sigma, c_0, \rho_0, \sigma_*)$ as solutions of the return task (1)–(4) respectively. Then

$$\|\sigma^{(1)} - \sigma^{(2)}\|^2_{H^1(0,l)} \le C\|g^{(1)} - g^{(2)}\|^2_{H^1(0,2l)}, \quad C = C(l, M_\sigma, c_0, \rho_0, \sigma_*),$$

where

$$f^{(j)} = (f_1^{(j)}, f_2^{(j)}, f_3^{(j)}),$$

$$f_1^{(j)}(x,t) = \frac{1}{2}\left[\left(g^{(j)}\right)'(t+x) - \left(g^{(j)}\right)'(t-x)\right], \quad f_2^{(j)} = -\frac{1}{\gamma}, \quad f_3^{(j)}(x) = -\frac{2\left(g^{(j)}\right)'(2x)}{\gamma}, \; j = 1, 2,$$

$$C = 3(l+1)\left(l + \frac{2}{\gamma^2}\right)\left[c_0^2\rho_0^2 + lM_\sigma^2\right]\exp\left\{\frac{2lM_\sigma^2}{\sigma_*^2}\right\} \times \left(15 + \frac{50M_\sigma^2}{\gamma^2\sigma_*^2} + \frac{5lM_\sigma^2}{\sigma_*^2}\left(1 + \frac{5M_\sigma^2}{\gamma^2\sigma_*^2}\right)\exp\left\{\frac{lM_\sigma^2}{2\sigma_*^2}\right\}\right) \times$$

$$\times \exp\left\{\left(3l + 5\beta + \frac{25l}{\gamma^2}\right)\frac{M_\sigma^2}{\sigma_*^2} + \left(\frac{l^2}{\gamma^2} + \frac{25l^2M_\sigma^4}{2\gamma^2\sigma_*^4} + \frac{10l\beta}{\gamma^2}\right)\exp\left\{\frac{lM_\sigma^2}{2\sigma_*^2}\right\} + \right.$$

$$\left. + \left(10\beta l\frac{M_\sigma^4}{\sigma_*^4} + 5l\beta + \frac{70l\beta}{\gamma^2}\right)\exp\left\{\frac{2lM_\sigma^2}{\sigma_*^2}\right\} + \frac{55l^2\beta M_\sigma^2}{\gamma^2\sigma_*^2}\exp\left\{\frac{5lM_\sigma^2}{2\sigma_*^2}\right\}\right\}.$$

Conclusions

For the proof of a conditional correctness of the considered task, it is proved the theorem where unlike the similar theorem in [3] in the above-stated theorem at a conclusion of the demanded constant in the main inequality not the assessment of a vector of q, but each estimates of its component was used q_1, q_2, q_3. In work [3] in calculations the norm of everyone components was estimated through norm of a vector of q as the assessment of norm of a vector of q is the sum of estimates its component. And in this work as it is told, above, estimates of everyone components were already used directly.

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